Optimal Foraging Theory Revisited

Masters Thesis Defense and Doctoral Qualifier Examination*

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June 5, 2007

* Typeset with LaTeX using `powerdot`.

Overview

Introduction

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- Goal

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- Goal: Discover the physics of high-level control

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- How to model?

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- How to measure performance?

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- High-level actuators are behaviors

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- Goal: Discover the physics of high-level control
- How to model?
- How to measure performance?
- High-level actuators are behaviors
- Behavioral ecology replaces physics

Solitary Agent Model

OFT Renewal Cycle

Figure 1: Classical OFT Markov Renewal Process

OFT Renewal Cycle

Figure 1: Classical OFT Markov Renewal Process

Parameters and Decision Variables

OFT Renewal Cycle

Figure 1: Classical OFT Markov Renewal Process

Parameters and Decision Variables

n: Number of task types.

OFT Renewal Cycle

Figure 1: Classical OFT Markov Renewal Process

Parameters and Decision Variables

n: Number of task types.

λ_i: Poisson encounter rate with tasks of type i (encounters/second).

OFT Renewal Cycle

Figure 1: Classical OFT Markov Renewal Process

Parameters and Decision Variables

n: Number of task types.

λ_i: Poisson encounter rate with tasks of type i (encounters/second).

λ: Merged Poisson encounter rate with all tasks (encounters/second).

OFT Renewal Cycle

Figure 1: Classical OFT Markov Renewal Process

Parameters and Decision Variables

$$\boxed{\lambda = \sum_{i=1}^{n} \lambda_i}$$

OFT Renewal Cycle

Figure 1: Classical OFT Markov Renewal Process

Parameters and Decision Variables

c^s: Cost rate for searching (points/second).

OFT Renewal Cycle

Figure 1: Classical OFT Markov Renewal Process

Parameters and Decision Variables

c^s: Cost rate for searching (points/second).

$\frac{c^s}{\lambda}$: Average cycle search cost (points/encounter).

OFT Renewal Cycle

Figure 1: Classical OFT Markov Renewal Process

Parameters and Decision Variables

p_i: Probability of processing a task of type i.

OFT Renewal Cycle

Figure 1: Classical OFT Markov Renewal Process

Parameters and Decision Variables

p_i: Probability of processing a task of type i.

τ_i: Average time processing a task of type i (seconds).

OFT Renewal Cycle

Figure 1: Classical OFT Markov Renewal Process

Parameters and Decision Variables

$g_i(\tau_i)$: Average gain from processing a task of type i (points).

OFT Renewal Cycle

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Parameters and Decision Variables

$g_i(\tau_i)$: Average gain from processing a task of type i (points).

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OFT Renewal Cycle

Figure 1: Classical OFT Markov Renewal Process

Parameters and Decision Variables

$g_i(\tau_i)$: Average gain from processing a task of type i (points).

c_i: Average cost rate while processing a task of type i (points/second).

$g_i(\tau_i) - c_i\tau_i$: Average net gain from processing a task of type i (points).

OFT Renewal Cycle

Figure 1: Classical OFT Markov Renewal Process

Random Variables

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Random Variables

g: Gross processing gain from k^{th} cycle.

OFT Renewal Cycle

Figure 1: Classical OFT Markov Renewal Process

Random Variables

g: Gross processing gain from k^{th} cycle.

c: Processing cost from k^{th} cycle.

OFT Renewal Cycle

Figure 1: Classical OFT Markov Renewal Process

Random Variables

g: Gross processing gain from k^{th} cycle.

c: Processing cost from k^{th} cycle.

τ: Processing time from k^{th} cycle.

OFT Renewal Cycle

Figure 1: Classical OFT Markov Renewal Process

Random Variables

$\widetilde{G}_k$: Total net gain from k^{th} cycle.

$\widetilde{C}_k$: Total cost from k^{th} cycle.

$\widetilde{T}_k$: Total time from k^{th} cycle.

OFT Renewal Cycle

Figure 1: Classical OFT Markov Renewal Process

Random Processes

OFT Renewal Cycle

Figure 1: Classical OFT Markov Renewal Process

Random Processes

$$\boxed{\widetilde{G}^N \triangleq \sum_{k=1}^{N} \widetilde{G}_k}$$

$$\boxed{\widetilde{C}^N \triangleq \sum_{k=1}^{N} \widetilde{C}_k}$$

$$\boxed{\widetilde{T}^N \triangleq \sum_{k=1}^{N} \widetilde{T}_k}$$

OFT Renewal Cycle

Figure 1: Classical OFT Markov Renewal Process

Statistics

OFT Renewal Cycle

Figure 1: Classical OFT Markov Renewal Process

Statistics

$$\overline{g} \triangleq \mathrm{E}\left(g\right) = \sum_{i=1}^{n} \frac{\lambda_i}{\lambda} p_i g_i(\tau_i)$$

$$\overline{c} \triangleq \mathrm{E}\left(c\right) = \sum_{i=1}^{n} \frac{\lambda_i}{\lambda} p_i c_i \tau_i$$

$$\overline{\tau} \triangleq \mathrm{E}\left(\tau\right) = \sum_{i=1}^{n} \frac{\lambda_i}{\lambda} p_i \tau_i$$

OFT Renewal Cycle

Figure 1: Classical OFT Markov Renewal Process

Statistics

$$\operatorname{var}(g-c) = \sum_{i=1}^{n} \frac{\lambda_i}{\lambda} p_i (g_i(\tau_i) - \overline{g})^2$$

OFT Renewal Cycle

Figure 1: Classical OFT Markov Renewal Process

Statistics

$$\mathrm{E}\left(\widetilde{G}^N\right) = N\,\mathrm{E}\left(\widetilde{G}_1\right) = N\left(\overline{g} - \overline{c} - \frac{c^s}{\lambda}\right)$$

$$\mathrm{E}\left(\widetilde{C}^N\right) = N\,\mathrm{E}\left(\widetilde{C}_1\right) = N\left(\overline{c} + \frac{c^s}{\lambda}\right)$$

$$\mathrm{E}\left(\widetilde{T}^N\right) = N\,\mathrm{E}\left(\widetilde{T}_1\right) = N\left(\overline{\tau} + \frac{1}{\lambda}\right)$$

OFT Renewal Cycle

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Statistics

$$\mathrm{var}\left(\widetilde{G}^N\right) = N\,\mathrm{var}\left(\widetilde{G}_1\right) = N\left(\mathrm{var}\left(\overline{g}-\overline{c}\right) + \left(\frac{c^s}{\lambda}\right)^2\right)$$

OFT Renewal Cycle

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Finite Lifetime?

Processing-Only Renewal Cycle

Figure 2: Finite Task Lifetime Markov Renewal Process

Processing-Only Renewal Cycle

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Split Poisson Parameters

Processing-Only Renewal Cycle

Figure 2: Finite Task Lifetime Markov Renewal Process

Split Poisson Parameters

λ_i^p: Split Poisson encounter rate with processed tasks of type i (encounters/second).

Processing-Only Renewal Cycle

Figure 2: Finite Task Lifetime Markov Renewal Process

Split Poisson Parameters

λ_i^p: Split Poisson encounter rate with processed tasks of type i (encounters/second).

$$\boxed{\lambda_i^p \triangleq p_i \lambda_i}$$

Processing-Only Renewal Cycle

Figure 2: Finite Task Lifetime Markov Renewal Process

Split Poisson Parameters

λ_i^p: Split Poisson encounter rate with processed tasks of type i (encounters/second).

λ^p: Merged Split Poisson encounter rate with all processed tasks (encounters/second).

$$\boxed{\lambda_i^p \triangleq p_i \lambda_i}$$

Processing-Only Renewal Cycle

Figure 2: Finite Task Lifetime Markov Renewal Process

Split Poisson Parameters

λ_i^p: Split Poisson encounter rate with processed tasks of type i (encounters/second).

λ^p: Merged Split Poisson encounter rate with all processed tasks (encounters/second).

$$\boxed{\lambda_i^p \triangleq p_i \lambda_i} \qquad \boxed{\lambda^p \triangleq \sum_{i=1}^{n} \lambda_i^p}$$

Processing-Only Renewal Cycle

Figure 2: Finite Task Lifetime Markov Renewal Process

Random Variables

Processing-Only Renewal Cycle

Figure 2: Finite Task Lifetime Markov Renewal Process

Random Variables

g^p: Gross processing gain from k^{th} processing cycle.

c^p: Processing cost from k^{th} processing cycle.

τ^p: Processing time from k^{th} processing cycle.

Processing-Only Renewal Cycle

Figure 2: Finite Task Lifetime Markov Renewal Process

Random Variables

G_k: Total net gain from k^{th} processing cycle.

C_k: Total cost from k^{th} processing cycle.

T_k: Total time from k^{th} processing cycle.

Processing-Only Renewal Cycle

Figure 2: Finite Task Lifetime Markov Renewal Process

Random Processes

Processing-Only Renewal Cycle

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Random Processes

$$\boxed{G^{N^p} \triangleq \sum_{k=1}^{N^p} G_k}$$

$$\boxed{C^{N^p} \triangleq \sum_{k=1}^{N^p} C_k}$$

$$\boxed{T^{N^p} \triangleq \sum_{k=1}^{N^p} T_k}$$

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Processing-Only Renewal Cycle

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Statistics

Processing-Only Renewal Cycle

Figure 2: Finite Task Lifetime Markov Renewal Process

Statistics

$$\overline{g^p} \triangleq \mathrm{E}\left(g^p\right) = \sum_{i=1}^{n} \frac{\lambda_i^p}{\lambda^p} g_i(\tau_i)$$

$$\overline{c^p} \triangleq \mathrm{E}\left(c^p\right) = \sum_{i=1}^{n} \frac{\lambda_i^p}{\lambda^p} c_i \tau_i$$

$$\overline{\tau^p} \triangleq \mathrm{E}\left(\tau^p\right) = \sum_{i=1}^{n} \frac{\lambda_i^p}{\lambda^p} \tau_i$$

Processing-Only Renewal Cycle

Figure 2: Finite Task Lifetime Markov Renewal Process

Statistics

$$\operatorname{var}\left(g^p - c^p\right) = \sum_{i=1}^{n} \frac{\lambda_i^p}{\lambda^p} \left(g_i(\tau_i) - \overline{g^p}\right)^2$$

Processing-Only Renewal Cycle

Figure 2: Finite Task Lifetime Markov Renewal Process

Statistics

$$\mathrm{E}\left(G^{N^p}\right) = N^p\,\mathrm{E}\left(G_1\right) = N^p\left(\overline{g^p} - \overline{c^p} - \frac{c^s}{\lambda^p}\right)$$

$$\mathrm{E}\left(C^{N^p}\right) = N^p\,\mathrm{E}\left(C_1\right) = N^p\left(\overline{c^p} + \frac{c^s}{\lambda^p}\right)$$

$$\mathrm{E}\left(T^{N^p}\right) = N^p\,\mathrm{E}\left(T_1\right) = N^p\left(\overline{\tau^p} + \frac{1}{\lambda^p}\right)$$

Processing-Only Renewal Cycle

Figure 2: Finite Task Lifetime Markov Renewal Process

Statistics

$$\operatorname{var}\left(G^{N^p}\right) = N^p \operatorname{var}\left(G_1\right) = N^p \left(\operatorname{var}\left(\overline{g^p} - \overline{c^p}\right) + \left(\frac{c^s}{\lambda^p}\right)^2 \right)$$

Optimization

Importance of Rate in Classical OFT

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- Time minimizer

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- Net gain maximizer

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- Lifetime pressure

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$$\operatorname*{aslim}_{N\to\infty} \frac{\widetilde{G}^N}{\widetilde{T}^N} \qquad \lim_{N\to\infty} \mathrm{E}\left(\frac{\widetilde{G}^N}{\widetilde{T}^N}\right)$$

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Importance of Rate in Classical OFT

- Time maximizer when net gain is negative
- Time minimizer when net gain is positive
- Net gain maximizer
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$$\operatorname*{aslim}_{N\to\infty} \frac{\widetilde{G}^N}{\widetilde{T}^N} \qquad \lim_{N\to\infty} \mathrm{E}\left(\frac{\widetilde{G}^N}{\widetilde{T}^N}\right)$$

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- Time maximizer when net gain is negative
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$$\operatorname*{aslim}_{N\to\infty} \frac{\widetilde{G}^N}{\widetilde{T}^N} = \frac{\mathrm{E}\left(\widetilde{G}_1\right)}{\mathrm{E}\left(\widetilde{T}_1\right)} = \lim_{N\to\infty} \mathrm{E}\left(\frac{\widetilde{G}^N}{\widetilde{T}^N}\right)$$

Importance of Rate in Classical OFT

- Time maximizer when net gain is negative
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$$\operatorname*{aslim}_{N\to\infty} \frac{\widetilde{G}^N}{\widetilde{T}^N} = \frac{\mathrm{E}\left(G_1\right)}{\mathrm{E}\left(T_1\right)} = \lim_{N\to\infty} \mathrm{E}\left(\frac{\widetilde{G}^N}{\widetilde{T}^N}\right)$$

Rate Maximization in Classical OFT

Figure 3: Visualization of Classical OFT Rate Maximization

Rate Maximization in Classical OFT

$$\widetilde{\gamma} \triangleq \frac{\mathrm{E}(\widetilde{G}_1)}{\mathrm{E}(\widetilde{T}_1)}$$

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Rate Maximization in Classical OFT

$$\widetilde{\gamma} \triangleq \frac{\mathrm{E}(\widetilde{G}_1)}{\mathrm{E}(\widetilde{T}_1)}$$

$$\lambda = \lambda_1,\, p_1 = 1$$

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Rate Maximization in Classical OFT

$$\boxed{\begin{gathered} \mathrm{E}(\widetilde{G}_1) = g_1(\tau_1) \\ \mathrm{E}(\widetilde{T}_1) = \tau_1 + \frac{1}{\lambda} \\ \widetilde{\gamma} \triangleq \frac{\mathrm{E}(\widetilde{G}_1)}{\mathrm{E}(\widetilde{T}_1)} \\ \lambda = \lambda_1, \, p_1 = 1 \end{gathered}}$$

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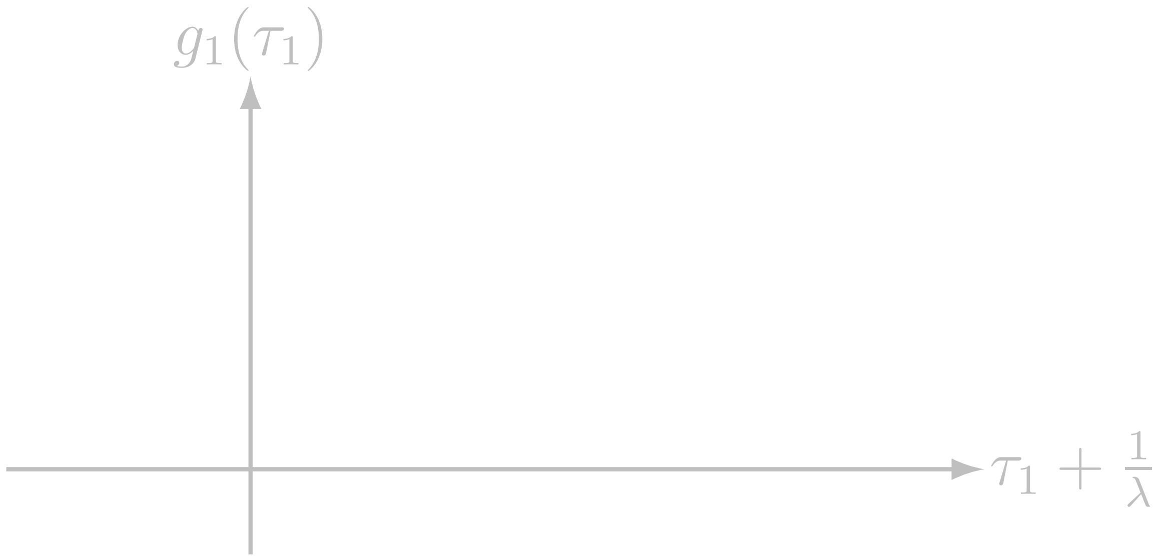

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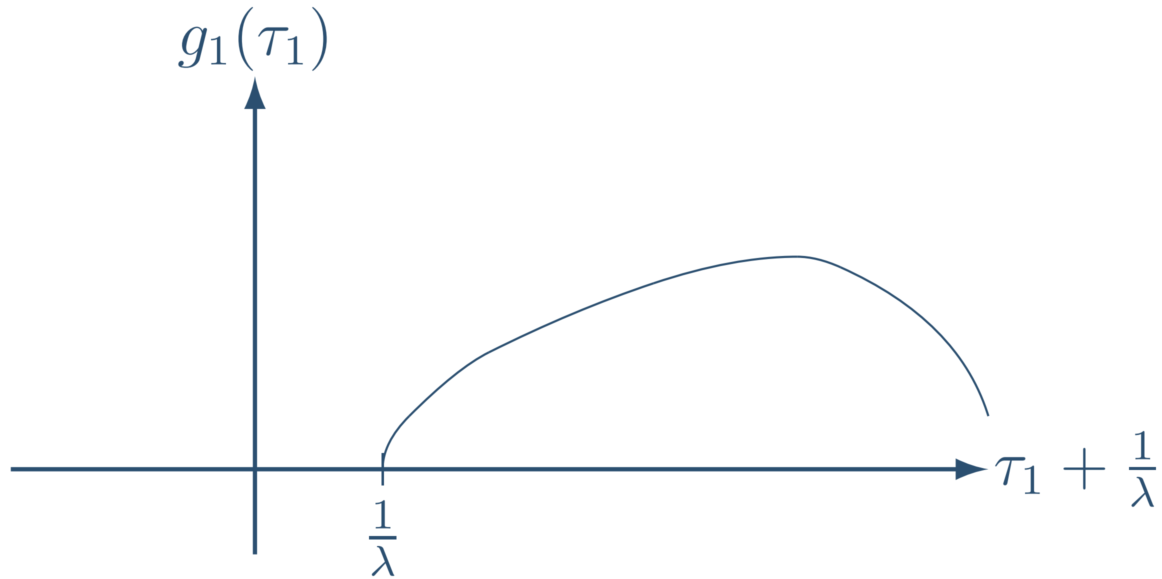

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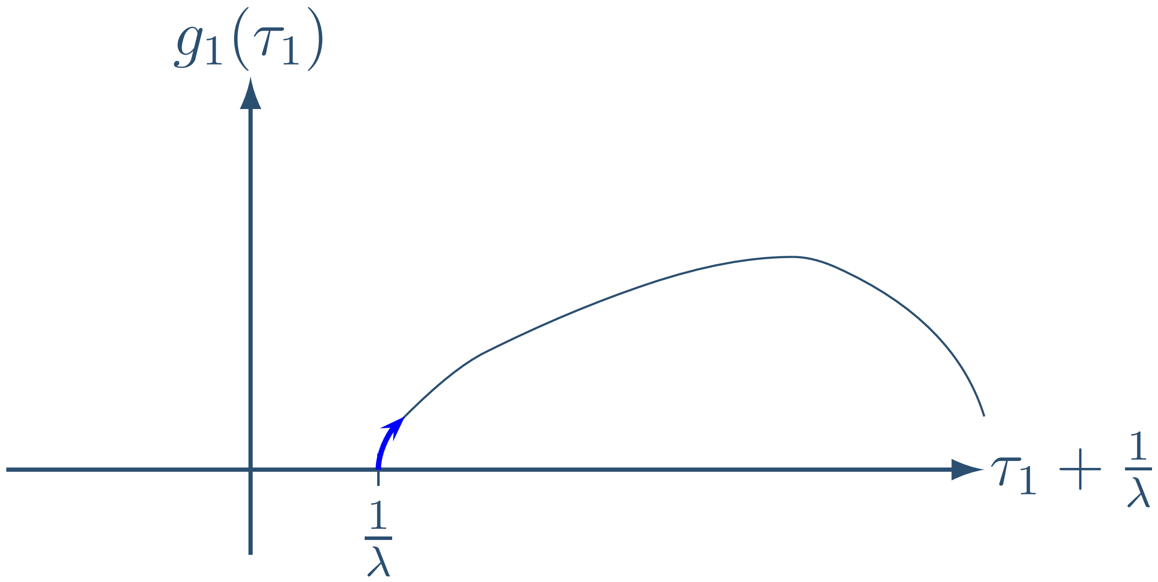

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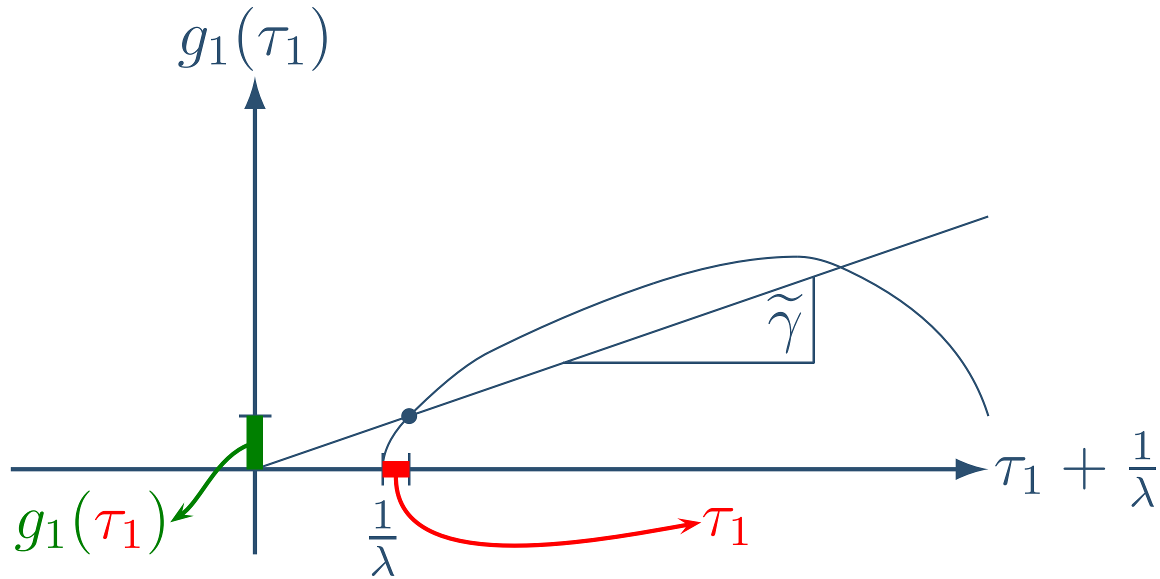

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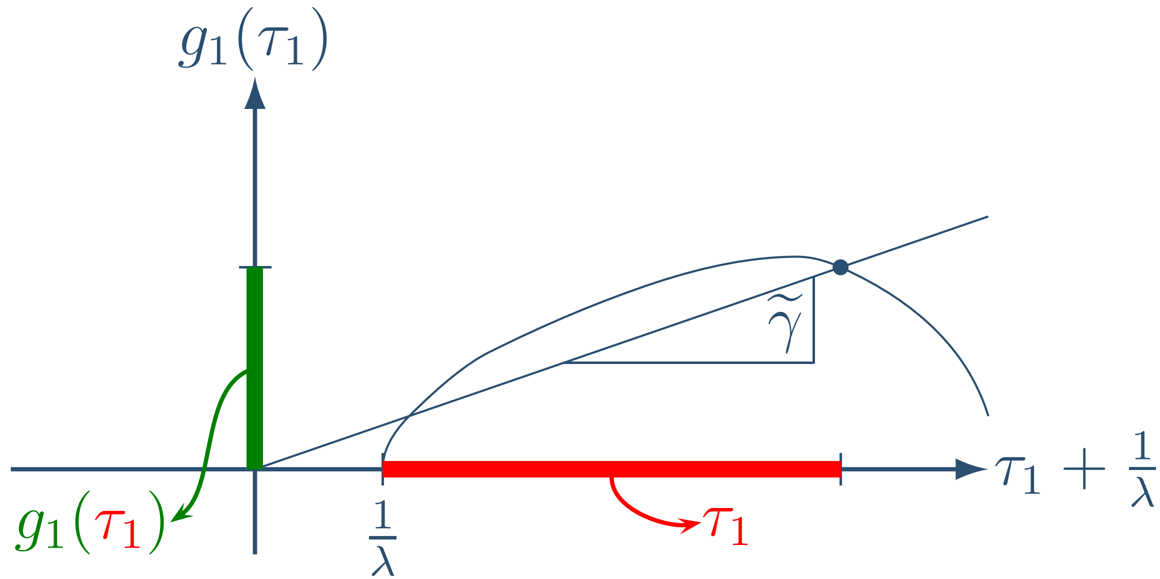

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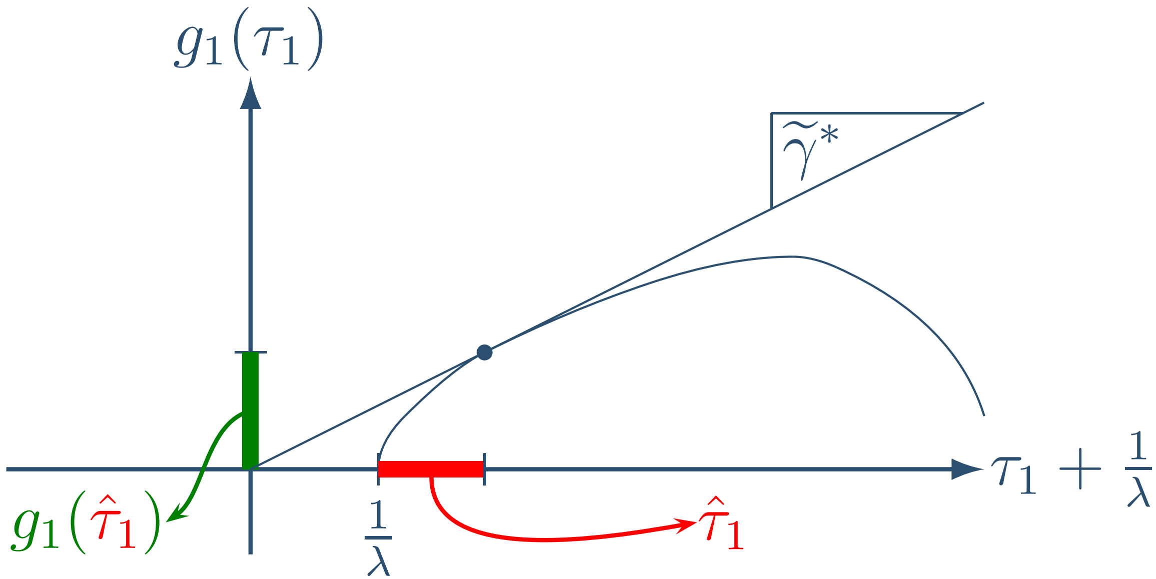
$$\lambda = \lambda_1,\ p_1 = 1$$
$$\Diamond^* \triangleq \max\{\Diamond\}$$



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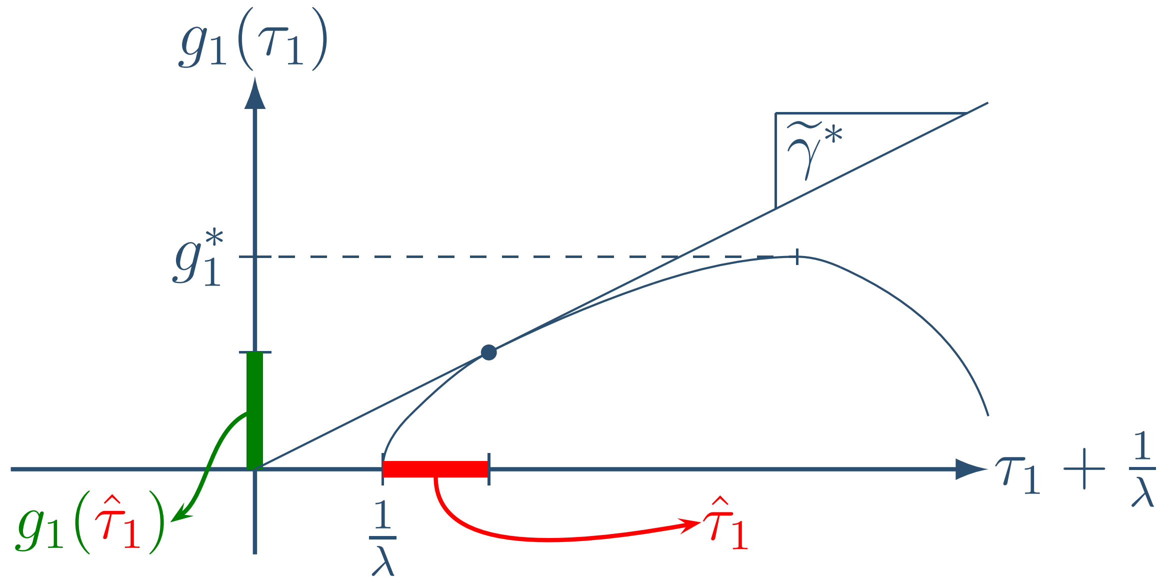

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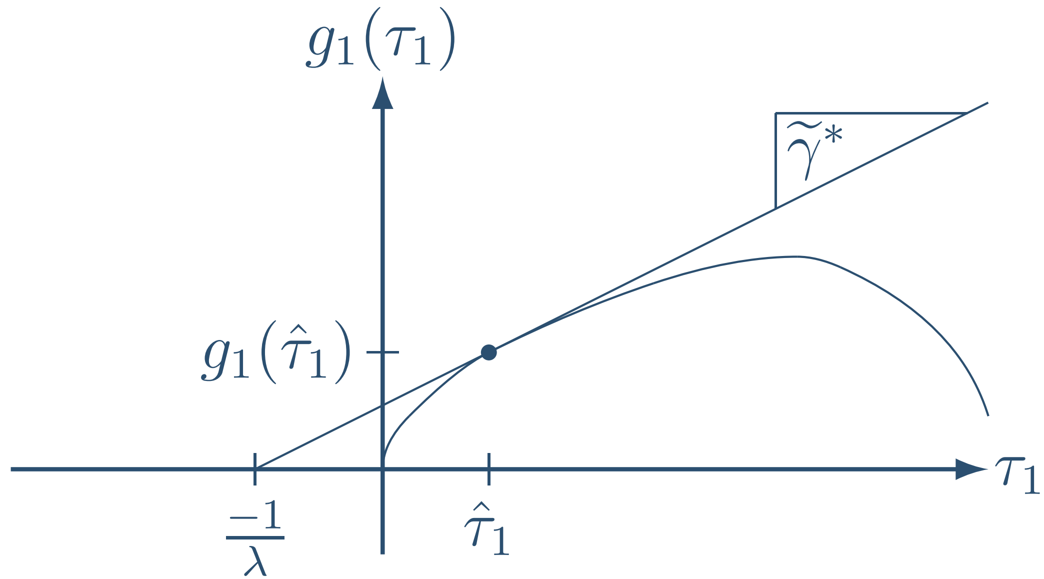

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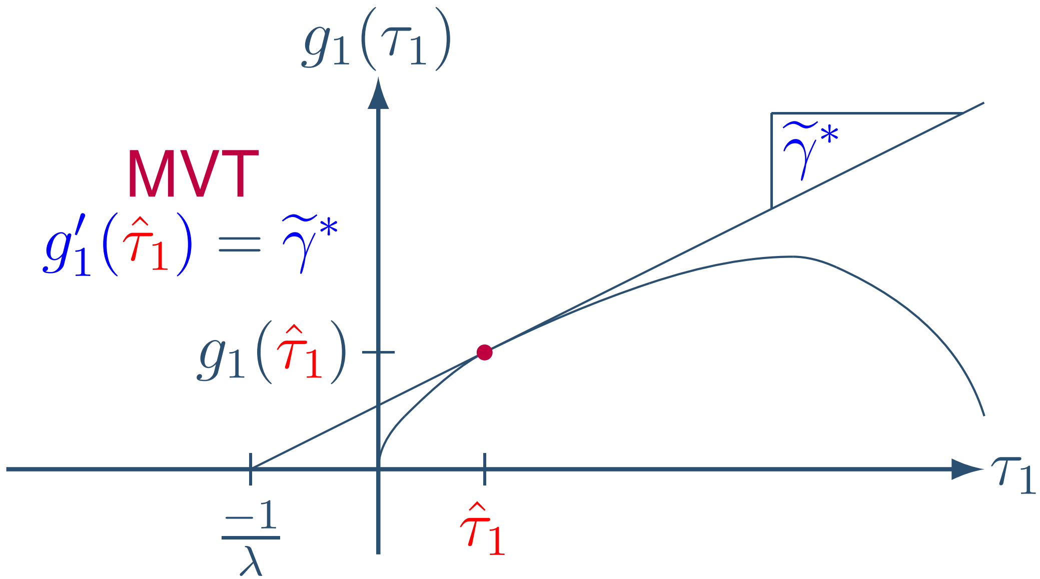

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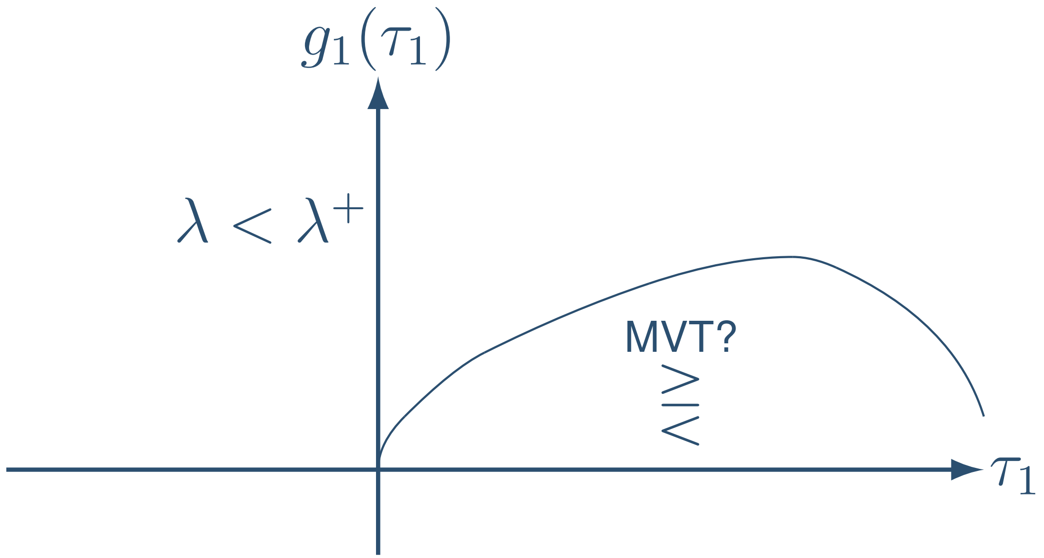
Rate Maximization in Classical OFT

$$\boxed{\begin{gathered} \mathrm{E}(\widetilde{G}_1) = g_1(\tau_1) \\ \mathrm{E}(\widetilde{T}_1) = \tau_1 + \tfrac{1}{\lambda} \\ \widetilde{\gamma} \triangleq \frac{g_1(\tau_1)}{\tau_1 + \frac{1}{\lambda}} \\ \lambda = \lambda_1,\, p_1 = 1 \\ \Diamond^* \triangleq \max\{\Diamond\} \end{gathered}}$$



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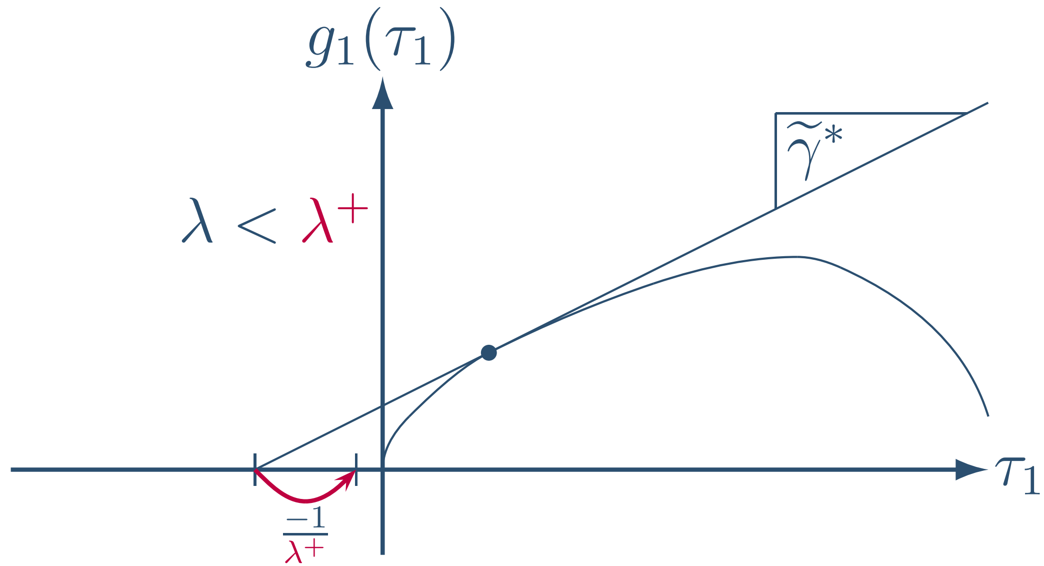

Figure 3: Visualization of Classical OFT Rate Maximization

$$
\begin{gathered}
\mathrm{E}(\widetilde{G}_1) = g_1(\tau_1) \\
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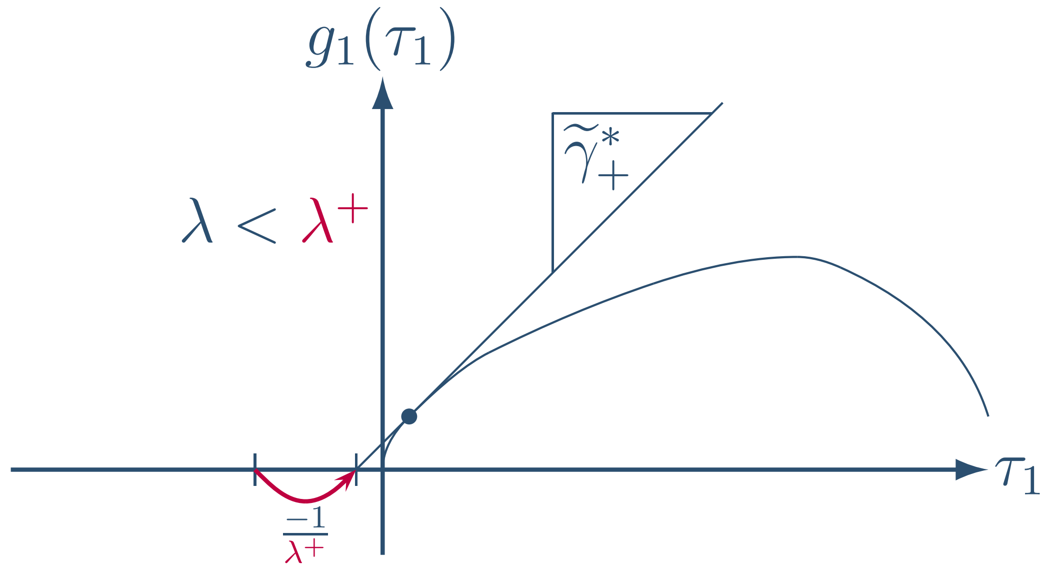
\widetilde{\gamma} \triangleq \frac{g_1(\tau_1)}{\tau_1 + \frac{1}{\lambda}} \\
\lambda = \lambda_1,\, p_1 = 1 \\
\Diamond^* \triangleq \max\{\Diamond\}
\end{gathered}
$$



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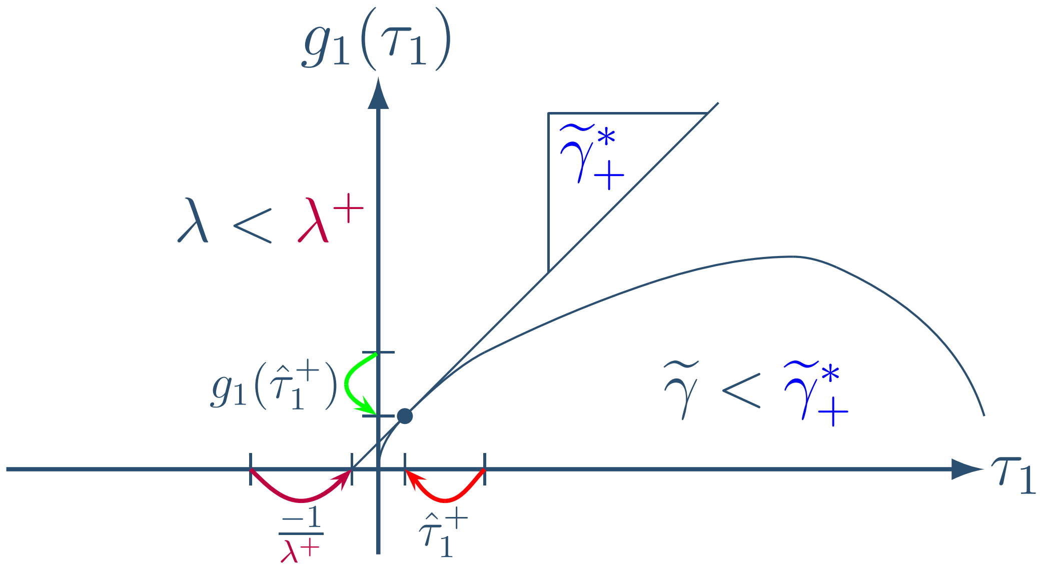

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Rate Maximization in Classical OFT

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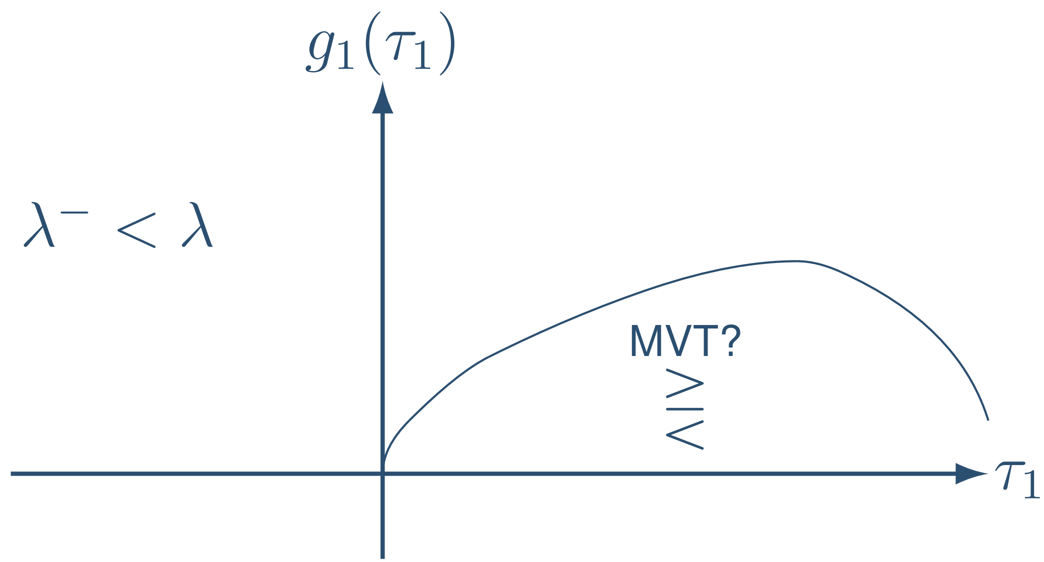

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$$\widetilde{\gamma} \triangleq \frac{g_1(\tau_1)}{\tau_1 + \frac{1}{\lambda}}$$
$$\lambda = \lambda_1,\, p_1 = 1$$
$$\lozenge^* \triangleq \max\{\lozenge\}$$

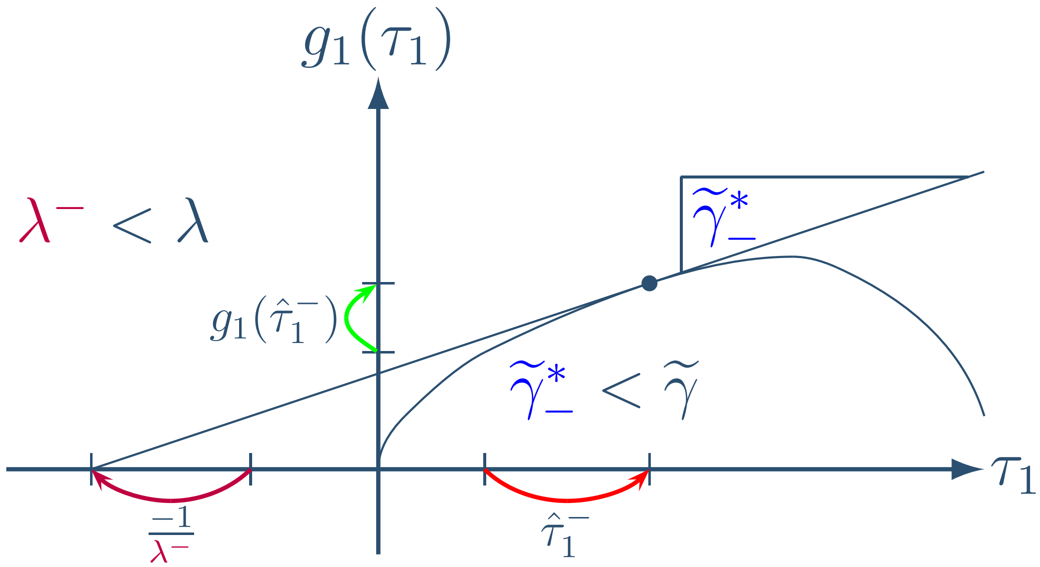

Figure 3: Visualization of Classical OFT Rate Maximization

$$\widetilde{\gamma} \triangleq \frac{\mathrm{E}(\widetilde{G}_1)}{\mathrm{E}(\widetilde{T}_1)}$$

Figure 4: Visualization of Generalized OFT Rate Maximization

Efficient OFT Rate Maximization

$$\overline{\tau} \triangleq \sum_{i=1}^{n} \frac{\lambda_i}{\lambda} p_i \tau_i \qquad \overline{g} \triangleq \sum_{i=1}^{n} \frac{\lambda_i}{\lambda} p_i g_i(\tau_i) \qquad \overline{c} \triangleq \sum_{i=1}^{n} \frac{\lambda_i}{\lambda} p_i c_i \tau_i$$

$$\widetilde{\gamma} \triangleq \frac{\mathrm{E}(\widetilde{G}_1)}{\mathrm{E}(\widetilde{T}_1)}$$

Figure 4: Visualization of Generalized OFT Rate Maximization

Efficient OFT Rate Maximization

$$\overline{\tau} \triangleq \sum_{i=1}^{n} \frac{\lambda_i}{\lambda} p_i \tau_i \qquad \overline{g} \triangleq \sum_{i=1}^{n} \frac{\lambda_i}{\lambda} p_i g_i(\tau_i) \qquad \overline{c} \triangleq \sum_{i=1}^{n} \frac{\lambda_i}{\lambda} p_i c_i \tau_i$$

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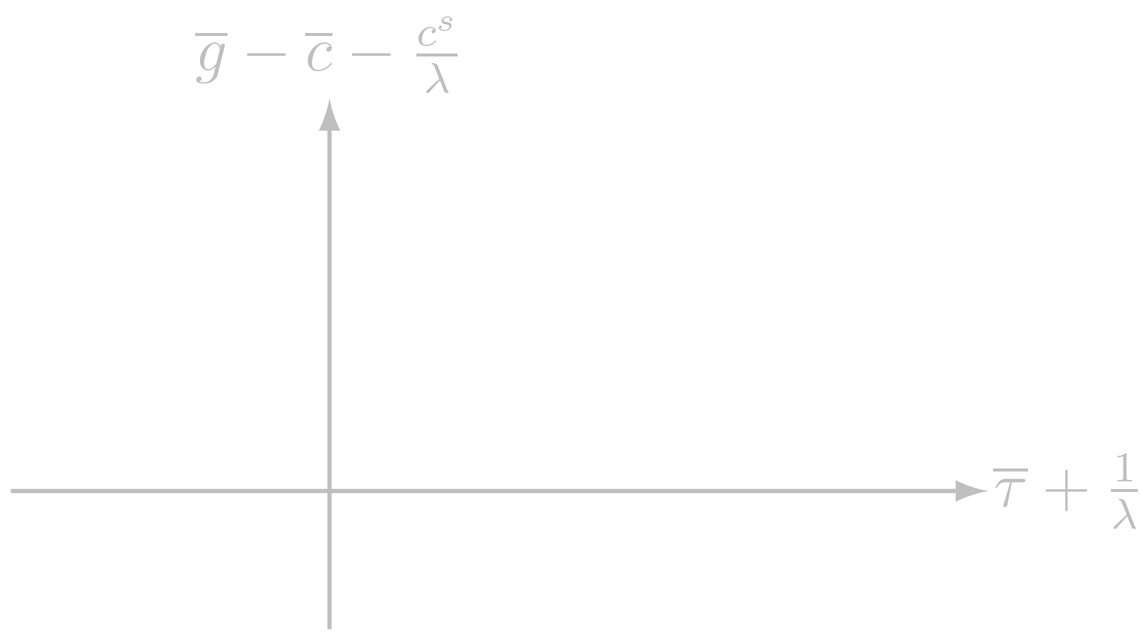

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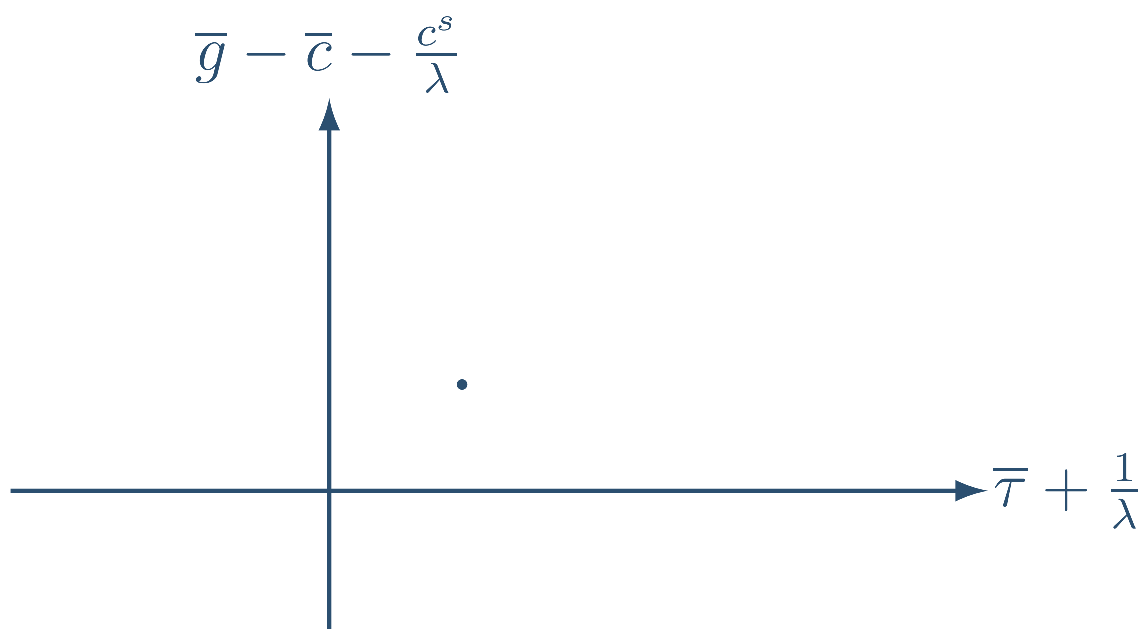

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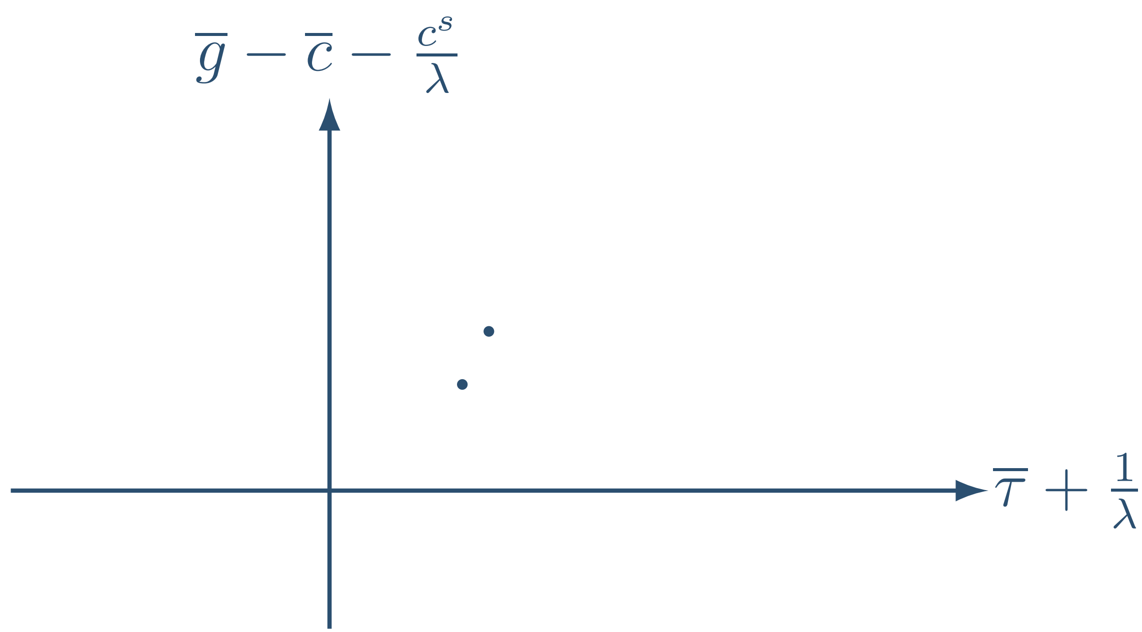

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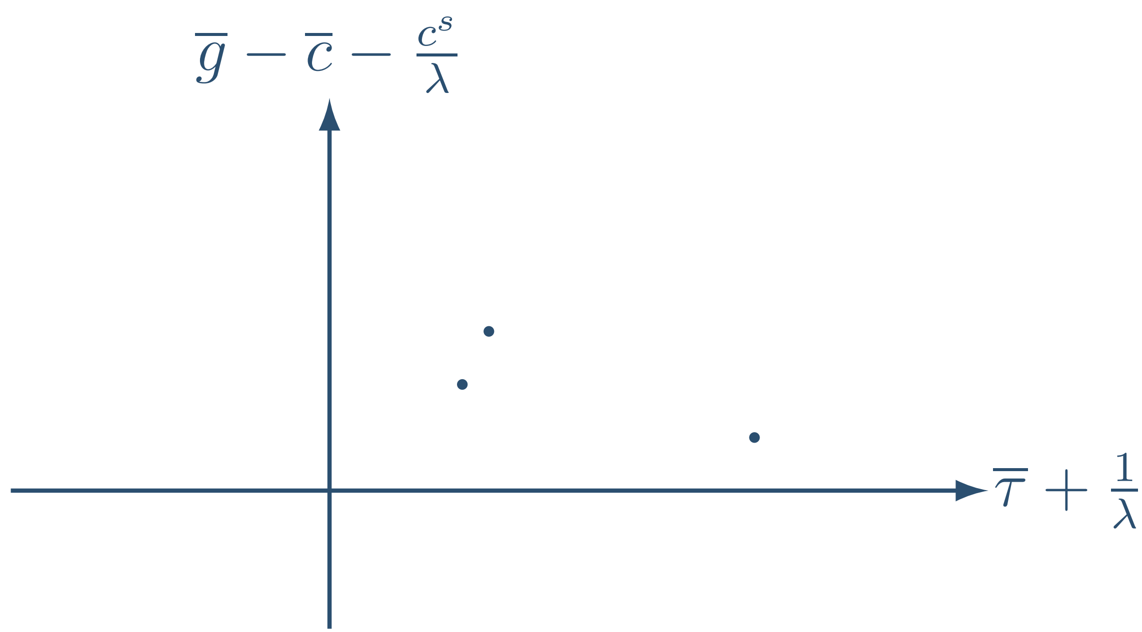

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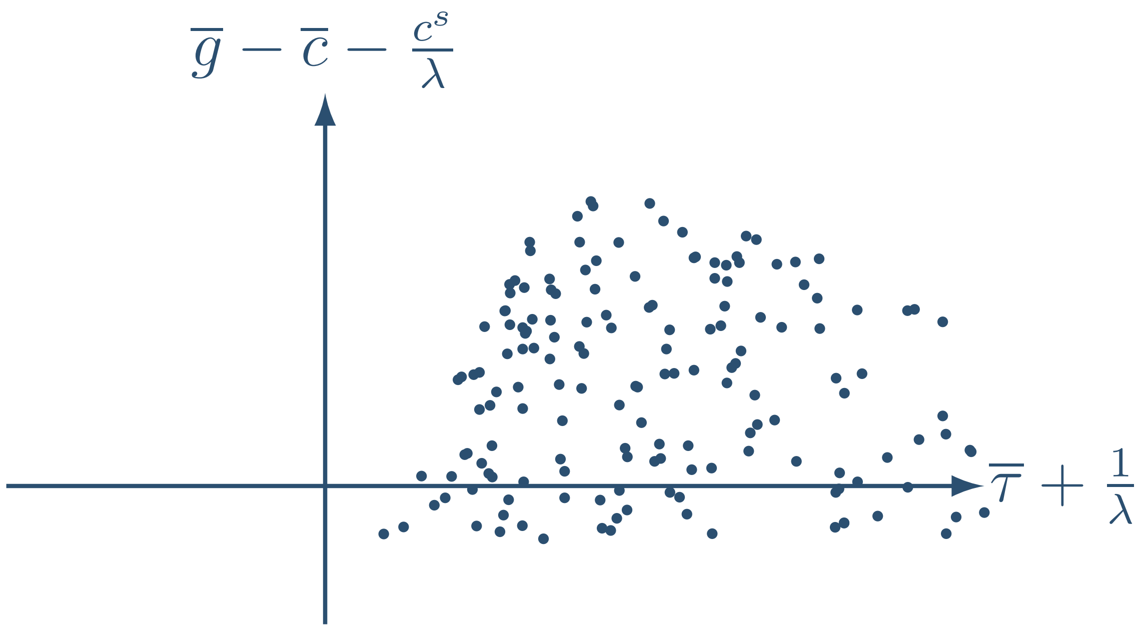

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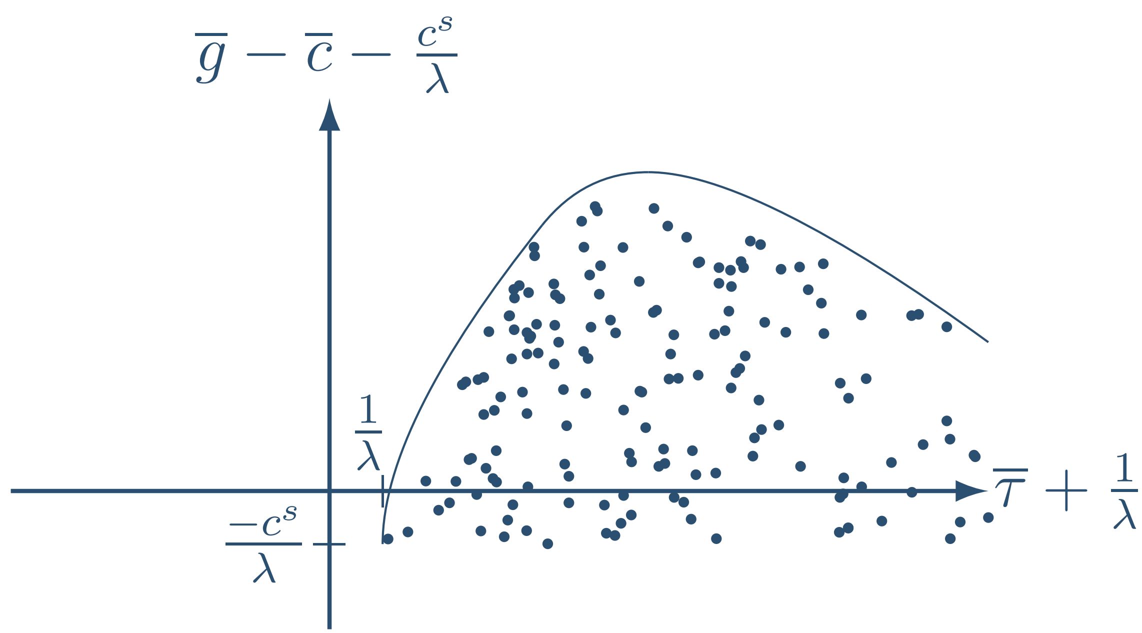

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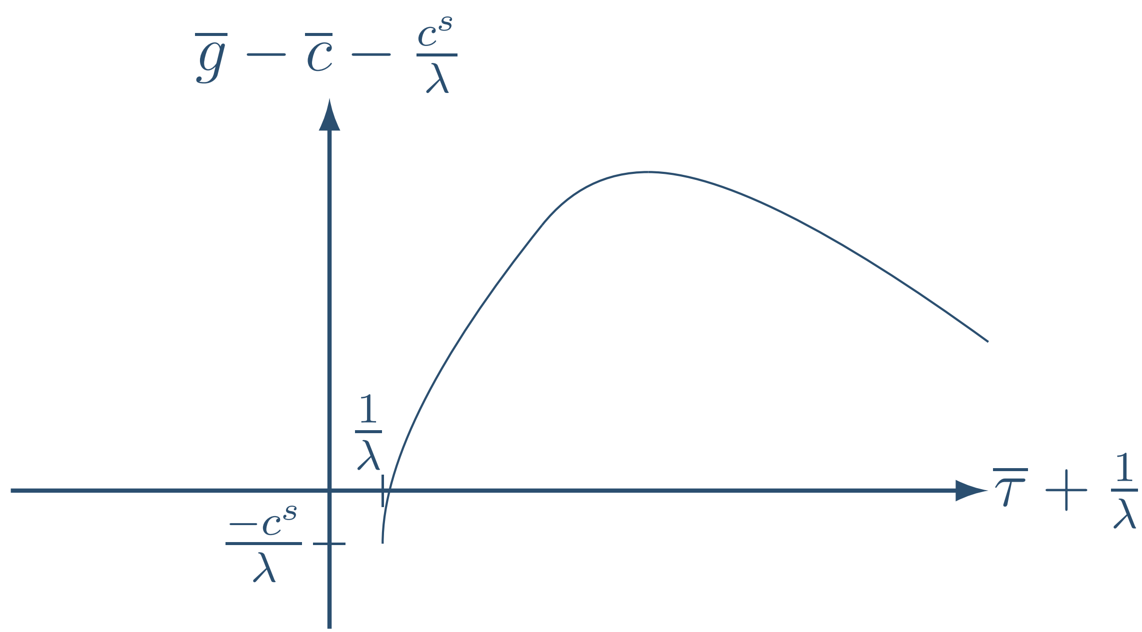

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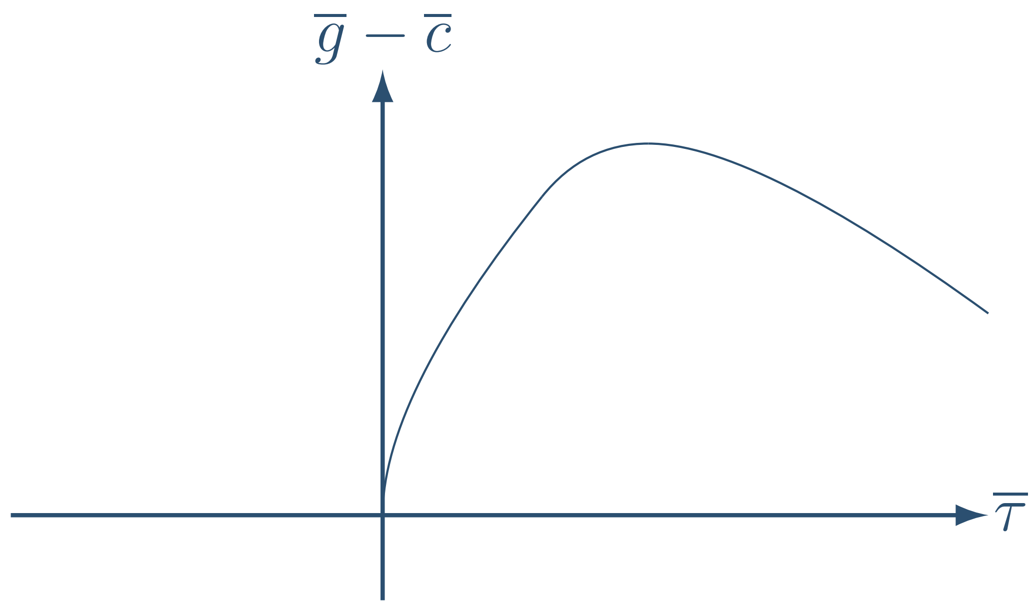

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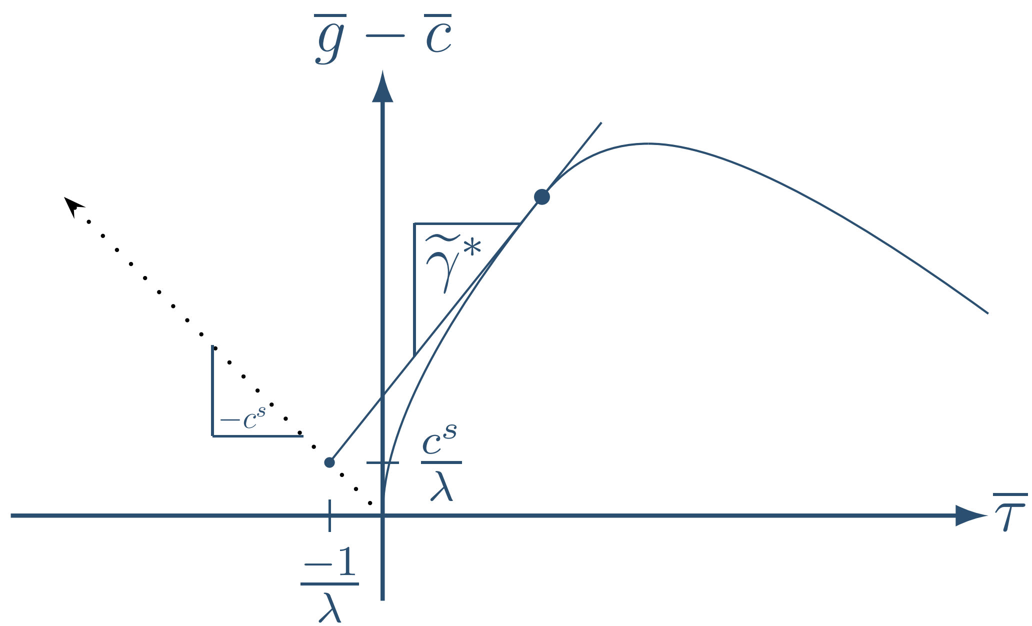

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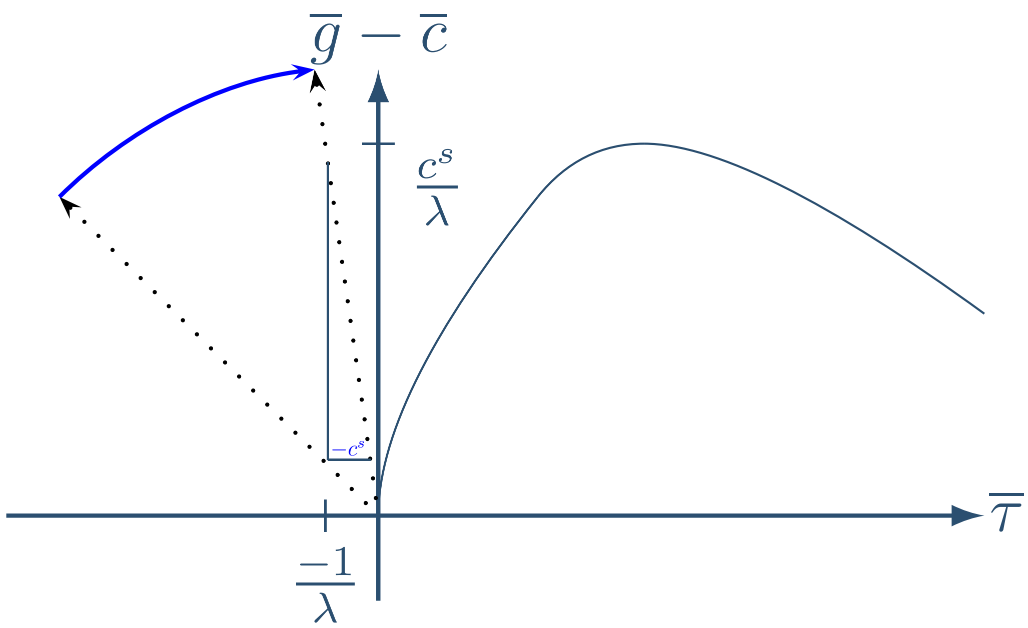

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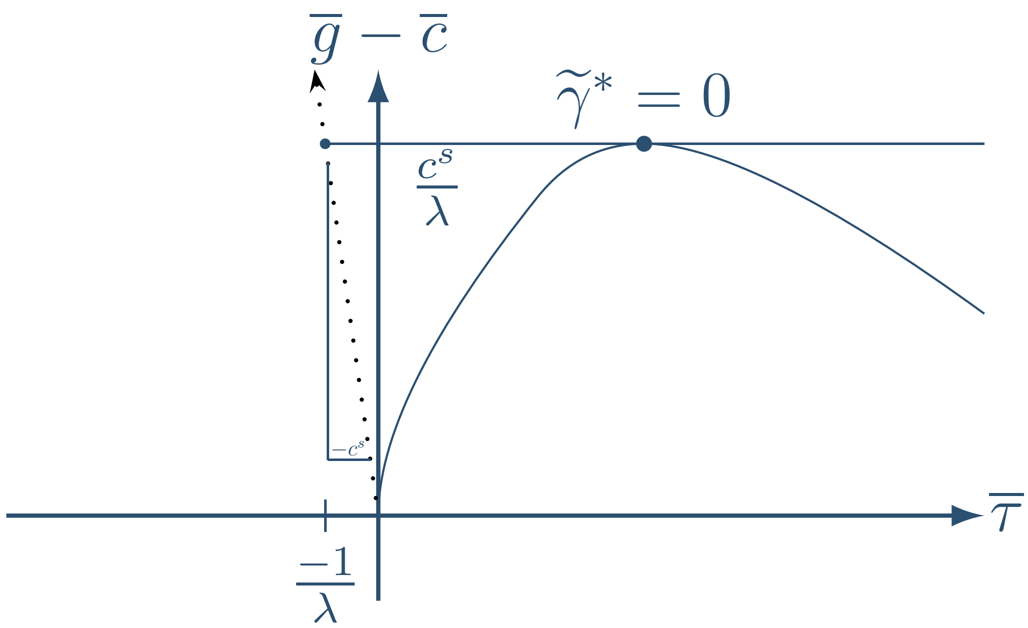

Figure 4: Visualization of Generalized OFT Rate Maximization

Finite-Lifetime Approach

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- Lifetime ends after N^p processed tasks

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Finite-Lifetime Rate Maximization

$$\mu \triangleq \mathrm{E}(G_1) = \overline{g^p} - \overline{c^p} - \frac{c^s}{\lambda^p}$$
$$\mathrm{E}(T_1) = \overline{\tau^p} + \frac{1}{\lambda^p}$$
$$\gamma \triangleq \frac{\mu - \frac{G^T}{N^p}}{\overline{\tau^p} + \frac{1}{\lambda^p}}$$
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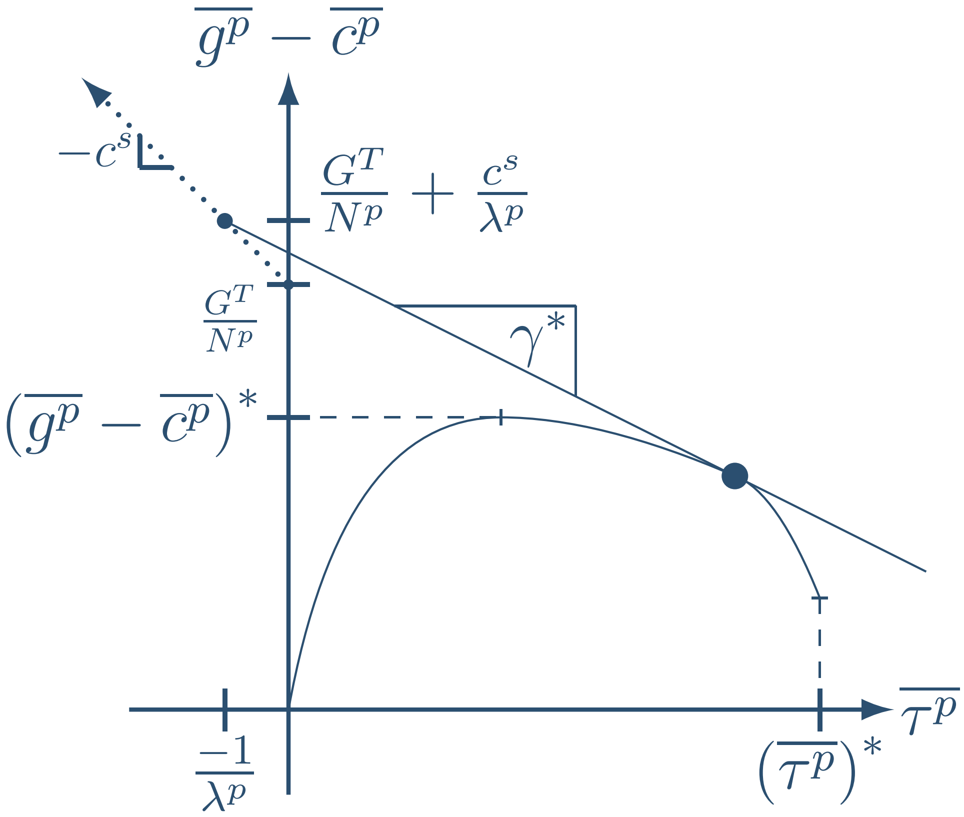

Figure 5: Excess Rate Maximization (N^p cycles and G^T threshold)

Reward-to-Variability Maximization

$$\mu \triangleq \mathrm{E}(G_1) = \overline{g^p} - \overline{c^p} - \frac{c^s}{\lambda^p} \qquad \sigma \triangleq \mathrm{std}(G_1) = \sqrt{\mathrm{var}\left(g^p - c^p\right) + \left(\frac{c^s}{\lambda^p}\right)^2}$$

$$\rho \triangleq \frac{\mu - \frac{G^T}{N^p}}{\sigma} \qquad \lozenge^* \triangleq \max\left\{\lozenge\right\}$$

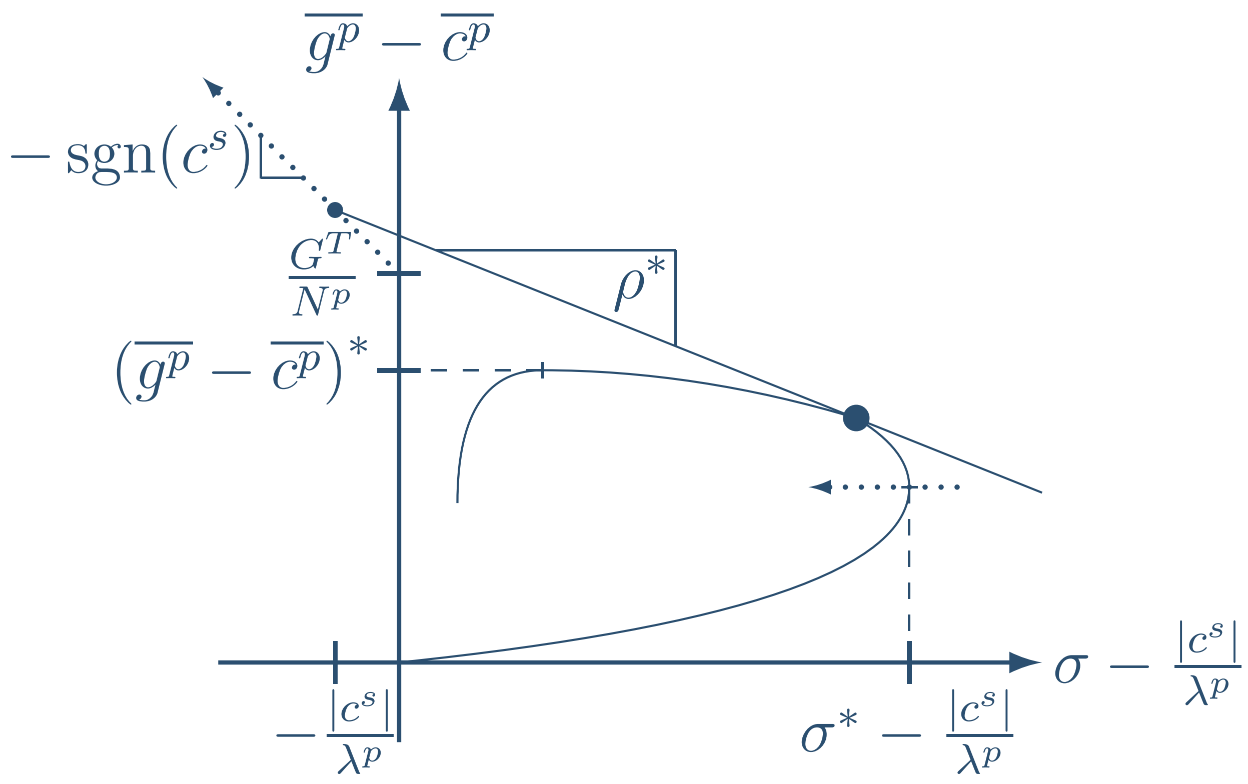

Figure 6: Sharpe Ratio Maximization (N^p cycles and G^T threshold)

Optimization by Discounting

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- Objectives A and B

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- Discount $w \in \mathbb{R}_{\geq 0}$

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$$\boxed{\mathrm{E}\left(\widetilde{G}_1\right) - \quad \mathrm{E}\left(\widetilde{T}_1\right)}$$

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Optimization by Discounting

- Objectives A and B
- Discount $\widetilde{\gamma}^* \in \mathbb{R}$
- Maximization of $A - \widetilde{\gamma}^* B$ is Pareto optimal
- Opportunity cost

$$\boxed{\mathrm{E}\left(\widetilde{G}_1\right) - \widetilde{\gamma}^* \mathrm{E}\left(\widetilde{T}_1\right)}$$

Optimization by Discounting

Other Possibilities

$$\mathrm{E}\left(\widetilde{G}_1\right) - \widetilde{w}_0\,\mathrm{E}\left(\widetilde{T}_1\right) \qquad \mathrm{E}\left(G_1\right) - w_0\,\mathrm{E}\left(T_1\right)$$

$$\mathrm{E}\left(\widetilde{G}_1\right) - \widetilde{w}_1\,\mathrm{std}\left(\widetilde{G}_1\right) \qquad \mathrm{E}\left(G_1\right) - w_1\,\mathrm{std}\left(G_1\right)$$

$$\mathrm{E}\left(\widetilde{G}_1 + \widetilde{C}_1\right) - \widetilde{w}_2\,\mathrm{E}\left(\widetilde{C}_1\right) \qquad \mathrm{E}\left(G_1 + C_1\right) - w_2\,\mathrm{E}\left(C_1\right)$$

Results

Advantage-to-Disadvantage Ratio

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Advantage-to-Disadvantage Ratio

$$A \triangleq a + \sum_{i=1}^{n} p_i a_i(\tau_i)$$

$$D \triangleq d + \sum_{i=1}^{n} p_i d_i(\tau_i)$$

$$J \triangleq \frac{A}{D}$$

Advantage-to-Disadvantage Ratio

Examples: Gain and Time Objectives

$$\frac{\mathrm{E}(G_1) - \frac{G^T}{N^p}}{\mathrm{E}(T_1)} = \frac{-c^s + \sum_{i=1}^{n} p_i \lambda_i \left(g_i(\tau_i) - c_i \tau_i - \frac{G^T}{N^p} \right)}{1 + \sum_{i=1}^{n} p_i \lambda_i \tau_i}$$

$$\mathrm{E}(G_1) - w\,\mathrm{E}(T_1) = \frac{-(c^s + w) + \sum_{i=1}^{n} p_i \lambda_i \left(g_i(\tau_i) - c_i \tau_i - w_i \tau_i \right)}{\sum_{i=1}^{n} p_i \lambda_i}$$

Advantage-to-Disadvantage Ratio

Examples: Efficiency-Type Objectives

$$\frac{\mathrm{E}(G_1) + \mathrm{E}(C_1) - \frac{G_g^T}{N^p}}{\mathrm{E}(C_1)} = \frac{\sum\limits_{i=1}^{n} p_i \lambda_i \left(g_i(\tau_i) - \frac{G_g^T}{N^p} \right)}{c^s + \sum\limits_{i=1}^{n} p_i \lambda_i c_i \tau_i}$$

$$\mathrm{E}(G_1) + \mathrm{E}(C_1) - w\,\mathrm{E}(C_1) = \frac{-wc^s + \sum\limits_{i=1}^{n} p_i \lambda_i \left(g_i(\tau_i) - w_i c_i \tau_i \right)}{\sum\limits_{i=1}^{n} p_i \lambda_i}$$

Constant Disadvantage Case

Assume that

- d_j is constant for all τ_j
- $\frac{a_j}{d_j}$ achieves its maximum at τ_j^*

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$$\frac{a_1(\tau_1^*)}{d_1(\tau_1^*)} > \frac{a_2(\tau_2^*)}{d_2(\tau_2^*)} > \cdots > \frac{a_{n-1}(\tau_{n-1}^*)}{d_{n-1}(\tau_{n-1}^*)} > \frac{a_n(\tau_n^*)}{d_n(\tau_n^*)}$$

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Exclude when

$$\frac{a + \sum_{i=1}^{k} a_i(\tau_i^*)}{d + \sum_{i=1}^{k} d_i(\tau_i^*)} > \frac{a_{k+1}(\tau_{k+1}^*)}{d_{k+1}(\tau_{k+1}^*)}$$

Generalized MVT

Assume that

$$\left(\frac{a_j}{d_j}\right)' < 0 \text{ for all } \tau_j$$

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Exclude when

$$\frac{a + \sum_{i=1}^{k} a_i(\tau_i^k)}{d + \sum_{i=1}^{k} d_i(\tau_i^k)} > \frac{a_{k+1}(0)}{d_{k+1}(0)}$$

Generalized MVT

where τ_j^k is found with

$$\frac{a_k'(\tau_j^k)}{d_k'(\tau_j^k)} = \frac{a + \sum_{i=1}^{k} a_i(\tau_i^k)}{d + \sum_{i=1}^{k} d_i(\tau_i^k)}$$

Generalized MVT

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This is a generalized version of the marginal value theorem.

Concluding Remarks

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- Provided analytical tools for finding optimal agent behavior
- Model can be expanded (e.g., recognition cost, behavior-dependent rates, nonlinear fuel costs)
- Optimization approaches can be enhanced using methods from post-modern portfolio theory
- Behaviors should be implemented on real agents

Questions?

Major Influences

- E.L. Charnov. Optimal foraging: The marginal value theorem. *Theoretical Population Biology*, 9(2): 129–136, April 1976.
- Eric L. Charnov and Gordon H. Orians. *Optimal foraging: some theoretical explorations*. PhD thesis, University of Washington, 1973.
- D.W. Stephens and J.R. Krebs. *Foraging Theory*. Princeton Univ. Press, Princeton, NJ, 1986.

Follow-Up

Applications

Applications

- Military
  - Surveillance
  - Offensive action

Applications

- Military
  - Surveillance
  - Offensive action
- Robotic Exploration (e.g., *DEPTHX*)
  - Deep Sea
  - Space

Applications

- Limited resources

Applications

- Limited resources
  - Retrieval space
  - Objects to deploy

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- Limited resources
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- Prioritization during real-time search

Applications

- Limited resources
  - Retrieval space
  - Objects to deploy
- Prioritization during real-time search
- Failure thresholds (e.g., G^T)

Applications

- Information foraging of human beings (i.e., analysis)

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  - Web behavior consistent with foraging predictions
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Future Directions

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  - Skew-sensitive objectives
  - Use of lower-partial moments

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- PMPT: Stochastic dominance

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  - Embraces risk sensitivity and a wider range of return distributions
- State-based dynamic programming methods
  - Modern approach
  - Improves performance
  - Increases behavioral complexity
  - Better machines and human-in-loop systems prevent complexity problems
  - Parameter estimation can be built-in

Influences for Future Work

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